Analysis of karst hydrodynamic behaviour using global approaches: a review

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\textbf{ABSTRACT}

This review-paper aims to give an overview of various existing methods for the analysis of hydrographs of karst springs and to give some critical points of view about interpretation of the results. The presented methods are the exponential reservoir methods, hyperbolic function approach, Mangin recession analysis and derived classification, the flow duration curves method and finally the simple and cross correlation and spectral methods. The existing critical literature shows that on the one hand, these methods are helpful for characterising the hydrographs and possibly for interpolating or extrapolating discharge in time. On the other hand, it appears that these methods are not very efficient to infer the structure of karst systems and to classify them because they are too strongly related to the frequency of the rainfall events.

\textbf{RESUME}

Cette note a pour but de donner un aperçu synthétique des méthodes les plus couramment utilisées pour l'analyse des hydrographes des sources karstiques. La description des méthodes est complétée de quelques critiques sur les interprétations généralement proposées. Les méthodes suivantes sont présentées: modèles à réservoirs exponentiels, modèle hyperbolique des courbes de récession, analyse des courbes de récession selon Mangin et classification dérivée, débits classés et finalement analyses corrélatoires et spectrales simple et croisée. La plupart de ces méthodes sont assez efficaces pour caractériser, interpoler, voir extrapololer les valeurs du débit des sources. Cependant, elles ne sont pas très adaptées pour inférer la structure des systèmes karstiques car elles dépendent beaucoup trop de la fréquence des événements pluvieux.

1. Introduction

This paper presents a review of a selection of some existing methods commonly used for analysing hydrographs of karst springs. It can be regarded as an introduction to the part on “Global concepts” of this special issue. Applications of these methods can be found in the next paper (LAROQUE et al., this issue) as well as in the references quoted. The present paper explains the principles of the methods and some of the major limitations which have to be considered if applied to the analysis of karst hydrodynamics.

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Prediction is an integral part of the management of karst water resources as well as of protection strategies against potential contamination, which karst aquifers are particularly subjected to. However, due to the large extent of the karst groundwater catchments, the large degree of heterogeneity, due to the high contrast in the hydraulic conductivity between conduit network and fissured matrix and particularly because of the scarcity of data available, prediction of hydraulic responses of karst has always posed a challenge to researchers and groundwater consultants.

However, as a result of their genetic history, karst groundwater systems have frequently one major advantage over other types of groundwater systems: flow of the total catchment is often focussed to one single spring and therefore allows an integral characterisation of the flow behaviour of the whole catchment just by measuring the outflow at a single point. This type of special feature and the lack of distributed data (hydraulic conductivity, variation in piezometric surface, etc.) spurred on the application of global or lumped parameter approaches to the hydrogeological description of karst systems. Furthermore, the relatively small proportion of the flow determining highly conductive karst conduits in the total aquifer volume makes them difficult to detect for parametrisation, thereby favouring the application of global models to spring discharge, where the influence of conduit flow on the total flow becomes apparent.

A very important attempt to build a conceptual model of karst aquifers was presented by Hobbs & Smart (1986). In this model (figure 1), three fundamental attributes describing the response of a limestone aquifer are identified, i.e. recharge, storage and transmission.
Recharge ranges between concentrated and dispersed end members (BURDON & PAPAKIS 1963), storage between high and low and/or saturated and unsaturated (includes subcutaneous storage). It can be argued that "saturated" and "unsaturated" cannot be regarded as a continuous variable because of the multiphase rock-water-air "unsaturated" system. In the light of the recent recognition of the importance of the unsaturated zone in the recharge process, it is essential to include both categories as discrete variables (MOHRLOK & SAUTER 1997). Finally, the groundwater flow is categorized on a scale that ranges from diffuse flow to conduit flow. As HOBBS & SMART (1986) pointed out, the three attributes are independent of each other, have different time scales and can therefore be distinguished. Recharge and unsaturated storage are treated separately and are accounted for a recharge calculation and its respective distribution over time.

Most of the methods commonly used for analyzing hydrographs of karst springs are derived from the time series analysis (MANGIN 1975, 1981a, 1981b, 1982, 1984, PADILLA & PULIDO-BOSCH 1995). Karst hydrologists use them for "interpreting" the response of karst springs, e.g. for inferring information about the characteristics of the three attributes described by HOBBS & SMART (1986). There is surprisingly not much critical literature discussing the application and the proposed interpretations of these methods. GRASSO & JEANNIN (1994) applied several methods to a well-characterized experimental field, where the karstic network is known along a significant part of the water catchment. They could compare inferences provided by hydrographs analysis with the known structure of the field. In the same paper, artificial time series (simple mathematical functions) are used for testing the simple correlation methods. This study shows that the interpretations derived from these methods are never unique. For instance, rain frequency or the size of the catchment can significantly increase the memory effect, although supposed to exclusively depend on the storage of the system.

EISENLOHR et al. (1997a and 1997b) reach the same conclusion by analyzing discharge time series generated by numerical models. In those models, there is no uncertainty as far as the hydraulic characteristics of the system are concerned. This paper shows that the karst network density also plays a role in the length of the memory effect. The influence of the recharge events frequency appears to be significant and may lead to misinterpret the respective significance of the storage when systems from different climatic regions are compared.

GRASSO (1998) presents an interesting chapter on a critical study of cross correlation methods applied to the analysis of karst system.

These analyses, as well as attempts to apply unit hydrograph methods to karst systems (GRASSO & JEANNIN 1994) show that karst systems are neither linear and nor stationary, especially when considering total precipitation as input and discharge as output. According to HOBBS & SMART (1986), they cannot be considered as one single system with two parallel subsystems (storage and drainage), but rather as a cascade of at least three subsystems having storage and transmissive characteristics each. Therefore, the system cannot be characterized by one single transfer function. This makes inferences on the structure of the system rather difficult.
The present paper shows the basics of the most common methods used for the hydrographs analysis of karst springs and point out the limitations to the interpretation of these methods when considering HOBBS & SMART (1986) conceptual model.

2. Methods commonly used in karst hydrology

2.1 Single event analysis

After a recharge event, any river/spring hydrograph can be decomposed into three components: the rising limb, the "rapid recession" and the "slow recession". Most of the existing methods aim to model the "rapid" and "slow recession" curves.

SIMPLE EXPONENTIAL RESERVOIR MODEL

It can be generally assumed that the rapid recession part is affected by the infiltration process and its temporal variability. It is superimposed on slow recession, which is assumed to reflect the characteristics of the aquifer considered as a large reservoir slowly releasing water. Discharge of such a reservoir is described by the classical "Maillet" equation:

\[ Q(t) = Q_0 \cdot e^{-\alpha t} \quad (\alpha > 0) \]  

This equation describes the change in discharge \( Q(t) \) with time \( t \) with \( Q_0 \) as the discharge along the straight line for \( t=0 \). The fitting is obtained by plotting the discharge against time on semi-logarithmic scale. For ideal conditions, i.e. single short term recharge event and an extended recession period, the hydrograph is approximated by a straight line with slope \(-\alpha\).

Interpretation of equation (1) yields an expression (eq. 2) for the assessment of the volume of the reservoir at time \( t_0 \).

\[ V = \int_{t_0}^{t} Q_0 \cdot e^{-\alpha t} \, dt \]  

This approach was used in a very large number of studies for volume estimation of groundwater resources (MANGIN 1975, PFAFF 1987, MARSAUD 1996, etc.). Assuming homogeneous aquifer and a simple shape catchment area, the approach can also be used for regional hydraulic parameter estimation (ATKINSON 1977, MILANOVIC 1981). This model includes the diffuse part of the storage subsystem of the HOBBS & SMART'S (1986) model. It is adequate for describing karst systems only during restricted periods of time (at low water stage) or to describe poorly karstified systems.

MULTIPLE EXPONENTIAL RESERVOIR MODEL

The structure of karst aquifers led FORKASIEWICZ & PALOC (1967) to assume that karst systems can be considered as consisting of several parallel reservoirs all contributing to the
discharge of the spring, each with its individual hydraulic characteristics. They suggested
the use of three reservoirs representing the low permeability fissured rock reservoir, the
conduit network respectively and the intermediate system.

The discharge $Q(t)$ at the spring is then described by

$$Q(t) = Q_{01} e^{-\alpha_1 t} + Q_{02} e^{-\alpha_2 t} + Q_{03} e^{-\alpha_3 t}$$

This equation is a sum of three terms each similar to equation (1). See this equation for the
meaning of the symbols and letters.

The volume of the reservoir can be assessed by a similar technique as in equation (2) (see
also Grasso & Jeannin 1994). Eisenlohr et al. (1997a and 1997b) analysed the
hydrographs issued from a finite element model. This shows that values of $\alpha_i$ cannot
exclusively be linked to the hydraulic conductivity of the various domains of the system
(FE-model in this case), but also depends on the structure of the permeability field in the
model. In particular, three exponential reservoirs can be fitted on the hydrograph of a
system containing only two classes of hydraulic conductivities.

Further, frequently, the different reservoirs, representing fissured, intermediate and conduit
systems are unrealistically considered as hydraulically isolated from each other. This means
that the parameter $\alpha_i$ obtained cannot be a characteristic of a specific sub-system. With
respect to the Hobbs & Smart (1986) model, the three subsystems are here not cascading
ones, but parallel and they do not include concentrated flow. The basic characteristics of
karst (conduit flow vs. diffuse flow) are not included in this model which can therefore only
provide “fictive” information about the structure of the system.

**HYPERBOLIC FUNCTION MODEL**

It is also possible to describe the spring discharge recession following a storm event by a
mathematical function such as:

$$Q(t) = \frac{Q_0}{(1 + \alpha \cdot t)^n}$$

Where $Q_0$ is the discharge at time $t=0$, which is the time when the recession is believed not
to be influenced anymore by the infiltration process (after the inflexion point). $\alpha$ and $n$ are
mathematical coefficients of the recession curve. This method allows a good fit of the reces­sion curves and can therefore be a good tool for modelling discharge through time. How­ever, because parameters $\alpha$ and $n$ cannot be related to physical parameters, it is not possible
to derive any physical information on the flow system.

**MANGIN APPROACH**

This approach consists of modelling a discharge recession curve using two different types
of functions: $\Psi(t)$ representing the contribution of the unsaturated zone and $\Theta(t)$ represent­ing the contribution of the saturated zone (Mangin 1975). Spring discharge can then be
approached by:

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\[ Q(t) = \Psi(t) + \Theta(t) \]

with

\[ \Theta(t) = Q_R \cdot e^{-\alpha t} \]

and

\[ \Psi(t) = q_0 \cdot \frac{1 - \eta t}{1 + \varepsilon t} \]

Where,

\( t_p \) = Time of the discharge peak
\( Q_{R_0} \) = Discharge of the baseflow at \( t_p \)
\( \eta \) = Parameter depending on the infiltration velocity, proportional to \( 1/t \) since the time when the recharge stops.
\( \varepsilon \) = Coefficient characterising the shape of the curve.
\( \alpha \) = Recession coefficient
\( t \) = Time
\( q_0 = Q_{t_p} - Q_{R_0} \) = Contribution of the infiltration water through the unsaturated zone at \( t_p \)

\( \alpha \)-values calculated with this model are comparable with those calculated with the exponential models. MANGIN (1975) suggests to assess the volume of the water that can be released from storage (or "dynamic volume") similarly as with an exponential model.

Based on the above method, MANGIN (1975) suggests to classify various types of karst systems. He considers the dynamic volume \( (V_d) \) as a characteristic of the saturated zone and compares this volume to the total volume of water discharged over an annual hydrological cycle \( (V_t) \). The ratio between \( V_d \) and \( V_t \) yields a parameter \( K \) called "regulation power":

\[ K = \frac{V_d}{V_t} \]

A second parameter, \( i \), is defined by MANGIN as being a characteristic of the recession curve. It is the value of the \( \Psi(t) \) function picked two days after the peak of the discharge curve. It is assumed to represent the importance of the retardation between infiltration and output.

For each spring, characteristic \( K \) and \( i \) values can be calculated and plotted in a \( K=f(i) \) diagram. Based on the position on this diagram, MANGIN (1975) defines 5 classes of karst springs (figure 2).

This method is frequently applied in France. This model is a simplified version of HOBBS & SMART (1986). In fact, it includes a single subsystem with two parallel components: a storage function \( \Theta(t) \) and a flow (transmission) function \( \Psi(t) \). The three attributes of HOBBS & SMART (1986) having different time scales are summarised by one single subsystem. We can therefore expect variations of the functions along time.

A critical study is presented by GRASSO & JEANNIN (1994). It points out that \( K \) appears to be quite stable at one spring for various years and events, but \( i \) is strongly dependant on the type of recharge. This produces a large spreading when various recessions from a spring are
*Figure 2: Classification of karst systems after Mangin (1975), based on K and i parameters. The dots in this diagram are various recessions from the same karst system (Milandrine, Jura Mountains, Switzerland). Parameter i strongly varies from one event to the other making classification difficult.*

plotted in a $K=f(i)$ diagram. Generally, the spreading extend over several of the identified classes. Therefore, the determination of the type of the system considered is highly subjective.

Further, the size of the catchment also have a strong effect on the value of parameter $i$, which cannot be considered as solely a characteristic of the unsaturated zone.

### 2.2 Time series analysis

Originally, these methods have been developed for analysing any time series, and later applied for flood forecasting in surface hydrology. MANGIN (1971, 1975, 1981a, 1981b, 1984) and PADILLA & PULIDO-BOSCH (1995) applied these techniques to the analysis of karst systems.

**FLOW DURATION CURVES**

Daily discharge values over a minimum of 1 year are ranked according to decreasing values of discharge. MANGIN (1971, 1975) suggests to model the distribution of the ranked
discharge using the following method. He first adjusts the scale of the discharge with the following relation:

\[ X = a \cdot \log \frac{Q}{Q_L} \]

where \( Q \) is a given discharge, \( Q_L \) is the lowest observed discharge. The factor \( a \) is an empirical factor for which \( a^+ \) is a measure of the variability of the log of discharge.

Mangin assumes that \( X \) can be modelled by a standard normal form of which we only consider the positive part. The relative percentage of the occurrence of a discharge lower than \( x \) is calculated using the following probability function:

\[ F(x) = \frac{2}{\sqrt{2\pi}} \int_{0}^{x} e^{-\frac{u^2}{2}} \, du \]

where \( u \) is the variable of the probability function and \( F(x) \) is the probability that \( X \) being smaller than \( x \).

A homogeneous system displays a linear response with slope \( a \) if \( F(x) \) is plotted against \( \log(Q/Q_L) \) in a quantile-quantile system. Derivations from linearity can be identified by a change in the slope of the straight line for a given range of discharge. An increase in the slope, for instance, can be interpreted as due to an increased storage within the system or to the existence of an overflow spring. A decrease of the slope can be related to an additional input of water to the system from other sources.

Beside numerous studies in France, examples of the application of this technique are provided in BONACCI (1987) and SOULIOS (1985). They identified special features in a karst system such as siphoning, overflow into neighbouring catchments and polje discharge.

This method is a good tool for pointing out such special behaviour. However, it does not provide any indication about the causes of the anomalies and therefore the interpretation has to be proposed based on the regional setting and on other indications.

**SIMPLE AUTOCORRELATION AND SPECTRUMS OF DISCHARGE TIME SERIES**

This method is a tool for identifying some overall characteristics (mainly cyclic variations) of time series, which are possibly not visible on a standard hydrograph. These characteristics may be related to structural characteristics of the flow system.

The basic principle is simply the comparison of a time series with itself. The two (same) time series are analysed by systematically increasing a relative shift between them. The sample correlation coefficient is calculated for each shift. This can be done in the time domain (correlograms) or in the frequency domain (spectrums).

The sample correlation is calculated as follows (BOX & JENKINS 1976):
\[ c_k = \frac{1}{N} \sum_{i=1}^{N-k} (z_i - \bar{z}) \cdot (z_{i+k} - \bar{z}) \]

\[ r_k = \frac{c_k}{c_0} \]

With:

- \( r_k \) is the sample autocorrelation function and \( c_k \) is the sample autocovariance function.
- \( k \) is the relative shift between the two (same) time series \( (0, 1, 2, \ldots, m) \).
- \( N \) is the number of observations in the time series.
- \( m \) is the maximum shift of the time series with respect to itself (usually \( m < N/3 \)).
- \( z_t \) is the \( t \)th realisation of the time series.
- \( z_{t+k} \) is the \( (t+k) \)th realisation of the time series.
- \( \bar{z} \) is the mean value of the \( N \) realisations of the time series.

The correlogram is the plot of \( r_k \) against \( k \). The correlogram represents a synthetic summary of the time series. It is sensitive to two characteristics: Large peaks in the time series will produce a large peak on the correlogram next to a shift of zero. Also, periodicities in the time series will give higher \( r_k \) values (secondary peaks) for the shift corresponding to the periodicity of the time series (e.g., annual cycles). Figure 3 shows an example of a fictitious time series and the corresponding correlogram. A white noise time series will give a “noise” correlogram displaying a low \( r_k \) for any \( k \) value.

The spectral analysis is similar but appears to be more powerful for evidencing periodicities within the time series. The sample spectrum function is given by the Fourier transform of the estimate of the autocovariance function. MANGIN (1982) suggest to use the following formulae:

\[ S(f) = 2 \left( 1 + 2 \sum_{k=1}^{m-1} D_k r_k \cos(2\pi f k) \right) \]

where:

- \( k \) is the relative shift between the two (same) time series \( (0, 1, 2, \ldots, m) \).
- \( f \) is the frequency \((f=\text{days}/2m \text{ for daily timestep), } \nu \geq f \geq 0\).
- \( r_k \) is the autocorrelation coefficient.
- \( D_k \) is a weighting function, \( D_k=(1+\cos(\pi km))/2 \).
- \( m \) is the maximum shift of the time series with respect to itself (usually \( m < N/3 \)).

MANGIN (1982) suggests to use the Tukey-Hanning weighting function \( (D_k) \) in order to amplify the peaks of the spectrum function. After this author, the periodicities appears more clearly by this way.
Figure 4: Fictitious time series and corresponding spectrum (from Grasso & Jeannin 1994). The spectrum makes periodicities of the time series very obvious.

Figure 4 provides an example of a fictitious time series and its spectrum. The three periodicities apparent in the time series are very well identified by the spectral analysis.

Mangin (1982) suggests to use the lag value for which the correlogram descends below 0.2 to define the "memory effect" of the system (see §2.3). He also uses the correlogram for describing the variations of the "reserves" and for classifying karst systems (see also §2.3).

This method has been applied mainly in France and nearby countries by several authors: Pulido-Bosch & Padilla (1988), Meus (1993), Mondain (1991), Marsaud (1996), etc.

Figure 3: Fictitious time series and corresponding correlograms (from Grasso & Jeannin 1994). Clearly, the shape of the correlogram depends on the breadth of the peaks of the hydrogram, but also on the frequency of the peaks. Note that correlograms are discrete graphs but for readability dots are not represented (1 dot per time unit in these particular cases).
From these various applications, one can observe that the time needed for the correlation to decrease below 0.2 considerably varies from one year to the next. This characteristic lag value strongly depends on the particular characteristics of the temporal recharge distribution, mainly the frequency and duration of storm events, but also, in a lesser extent, the depth of precipitation. Further, the size of the catchment has an important effect on this lag value. This characteristic dependency on the particular recharge distribution makes classifications difficult. The link between this "memory effect" and the "reserves" is also subject to discussion. More details about critical discussion can be found in Grasso & Jeannin (1994).

CROSS CORRELATION ANALYSIS OF PRECIPITATION AND DISCHARGE TIME SERIES

This type of analysis has also first been applied to karst systems by Mangin (1981a, 1981b, 1982, 1984). This author used statistical methods previously presented by Jenkins & Watts (1968) and Box & Jenkins (1976), who applied them to prediction and data completion.

This approach considers the system as a filter, which transforms the input time series into an output time series. As we will see at the end of this chapter, if the system is linear and stationary, a transfer function can be defined, which is a characteristic function of the system. The shape of the transfer function depends on the processes active within the system. Mangin (1982) suggests to link a particular shape of the transfer function to the structure of the flow system (figure 5). For this author, two features determine the shape of the transfer function: 1) the storage ("reserves"), which is responsible for the "memory" (mémoire) of the system, i.e. the "baseflow" of the transfer function, and 2) the "hierarchical drainage network", which determines the fast component of the flow, i.e. the peak of the transfer function. A presentation of the method with some theoretical and practical examples can be found in Padilla & Pulido-Bosch (1995).

The cross correlation coefficient function $r_{yk}$ and $r_{yk}$ are defined as:

$$r_{yk} = r_{yk}(k) = \frac{c_{xy}(k)}{s_x \cdot s_y} \quad \text{with} \quad c_{xy}(k) = \frac{1}{N} \sum_{i=1}^{N-k} (x_i - \bar{x}) \cdot (y_{i+k} - \bar{y})$$

$$r_{yk} = r_{yk}(k) = \frac{c_{yx}(k)}{s_x \cdot s_y} \quad \text{with} \quad c_{yx}(k) = \frac{1}{N} \sum_{i=1}^{N-k} (y_i - y) \cdot (x_{i+k} - \bar{x})$$

$$s_x^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2 \quad \text{and} \quad s_y^2 = \frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{y})^2$$

$c_{xy}$ is the cross covariance between input $(x)$ and output $(y)$ time series
$c_{yx}$ is the cross covariance between output $(y)$ and input $(x)$ time series
$k = \text{relative shift between the two time series (0, 1, 2, ..., m)}$
$N = \text{the number of observations in the time series}$
$m = \text{the maximum shift of the time series with respect to itself (usually } m < N/3).$
$x_i, y_i$ are the $i^{th}$ realisations of the respective time series
$x_{i+k}, y_{i+k}$ are the $(i+k)^{th}$ realisations of the respective time series
$\bar{x}$ and $\bar{y}$ are the mean values of the $N$ realisations of the respective time series
Types | Memory effect (Ri=0.2) | Spectral band breadth | Regulation time | Transfer function
---|---|---|---|---
Aliou | very low (5 days) | very high (0.30) | 10 to 15 days | ![Transfer function](image)
Baget | low (10-15 days) | high (0.20) | 20 to 30 days | ![Transfer function](image)
Fontestorbes | high (50-60 days) | low (0.10) | 50 days | ![Transfer function](image)
Torcal | very high (70 days) | very low (0.05) | 70 days | ![Transfer function](image)

Figure 5: Karst systems classification based on correlation and spectral analyses (from MANGIN 1982). After this author, any system having a similar transfer function to Aliou, Baget, Fontestorbes or Torcal type is supposed to have a similar structure to these systems.

If the input function can be assumed to be a random function, then the cross correlogram is equal to the transfer function which would be found by deconvoluting the output time series (spring discharge) by the input one (precipitation). Details about this relation between the correlogram and the transfer function can be found in NEUMAN & DE MARSILY (1976), DREISS (1989) or MANGIN (1984).

The cross spectrum is the Fourier transform of the cross correlation function. It is a complex number function of type (PADILLA & PULIDO-BOSCH 1995 derived from JENKINS & WATTS 1968):

$$S_C(f) = K_{xy}(f) - iQ_{xy}(f)$$

where

$$K_{xy}(f) = 2 \left( r_{xy}(0) + \sum_{k=1}^{m} \left( r_{xy}(k) + r_{yx}(k) \right) D_k \cdot \cos(2\pi f \cdot k) \right)$$

$$Q_{xy}(f) = 2 \sum_{k=1}^{m} \left( r_{xy}(k) - r_{yx}(k) \right) D_k \cdot \sin(2\pi f \cdot k)$$

with

- $D_k$ is a weighting function, $D_k=(1+\cos(k\pi m))/2$
- $f =$ frequency (1/day)
The cross spectrum can also be defined as (PADILLA & PULIDO-BOSCH 1995):

$$S_{xy}(f) = |A_{xy}(f)| \cdot e^{-i\theta_{xy}(f)}$$

with

$$|A_{xy}(f)| = \sqrt{K_{xy}^2(f) + Q_{xy}^2(f)}$$  \quad \text{Amplitude function}

$$\theta_{xy}(f) = \arctan\left(\frac{Q_{xy}(f)}{K_{xy}(f)}\right)$$  \quad \text{Phase function}

The amplitude function is a decomposition of the total covariance between input and output with respect to frequency. It is a qualitative indication of the way a certain input is transformed into an output function.

The phase function allows to assess the lag (in frequency) between input and output for any given frequency:

$$\varphi(f) = \theta_{xy}(f)/2\pi f$$

The combination between simple and crossed spectral analyses allows defining two further functions:

The coherence function:

$$\gamma_{xy}(f) = \frac{A_{xy}(f)\sqrt{S_x(f) \cdot S_y(f)}}{\sqrt{S_x(f) \cdot S_y(f)}}$$

and the gain function:

$$g_{xy}(f) = \frac{A_{xy}(f)\sqrt{S_x(f)}}{\sqrt{S_x(f)}}$$

with

$A_{xy} = \text{amplitude function}$

$S_x, S_y = \text{spectral density functions of } x \text{ and } y \text{ time series}$

The coherence function shows changes of the correlation between input and output with respect to frequency. It can be used for assessing the linearity of the system. The gain function shows how the input signal is amplified ($g_{xy}(f) > 1$) or reduced ($g_{xy}(f) < 1$) by the system. It is assumed to be linked to the significance of the storage, which is supposed to reduce high and middle frequency events, but increase low frequency ones.

Note that the definition of the coherence and gain functions may change from one author to the other, especially because these methods are used in many different scientific domains.

2.3 Karst systems classification based on correlation and spectral analysis

Correlation and spectral analysis are "objective" methods for classifying time series. MANGIN (1982, 1984) defined four parameters based on the correlation and spectral
analysis of discharge-discharge and precipitation-discharge time series in order to attempt to classify karst systems. This author defines them as follows:

1) **Memory effect**: It is calculated from the simple spectrum. It is the time (in days) needed for the discharge simple correlogram to reach a correlation value \( r_k \) lower than 0.2. Mangin assumes that this parameter only depends on the storage of the system.

2) **Spectral band breadth**: This is the frequency domain for which the system does not completely filter the input signals. Beyond this typical frequency, the output spectrum is equal to zero or can be considered as a white noise. The four functions defined above (phase, amplitude, coherence and gain) can only be interpreted within the range of the spectral band breadth.

3) **Regulation time**: It is obtained by dividing the maximum value of the simple spectrum function by the integral of this function between zero and infinite. It provides an information about the duration of the influence of a unitary input. It is assumed to give a qualitative indication of the global organisation of the flow system (conduit network) within the karst system.

4) **Transfer function**: It is approached by the cross correlation function if the input function can be considered as random. It is a relatively rough approximation of the unitary hydrograph. It is used for classifying karst systems into four classes (figure 5) ranging from a very peaky transfer function corresponding to well drained systems (weak change of inputs into outputs), to a very flat and retarded one corresponding to poorly drained systems (high filtering of inputs by the system).

Real systems are supposed to range between two extremities (figure 5): On the one end, well drained systems, which have low storage, then a low memory effect and a large spectral band, a short regulation time and a high and narrow transfer function. On the other end, inertial systems, which have a large memory effect, a narrow spectral band, a large regulation time and a flat transfer function. GRASSO (1998) shows that results of the cross correlation analysis are strongly influenced by the frequency of the input events (rainfall) and then that the transfer function can hardly be considered as a characteristic of a given karst system.


### 3. Discussion and conclusion

The three last methods presented in this paper are derived from the time series analysis. They have been developed for analysing the behaviour of a parameter along time (periodicity analysis, trend analysis, etc.) and can therefore allow some forecasting of the value of a parameter in time. They can be applied to karst systems for the same purpose.

In some cases, this type of analysis can give evidences of any special behaviour, which can be related to a particular structure of the flow system. Under ideal conditions (same input
signals) the cross correlation and spectrum methods can be used to compare and classify karst hydrological systems (Padilla & Pulido-Bosch 1995). Meanwhile, as rainfalls are always different over different catchments, the comparison of various systems is biased by the effect of different input time series (precipitation).

Further, time series analysis methods do not include physical processes linking space processes with time response. Therefore, any interpretation towards inferring a particular structure of a flow system is rather uncertain and hazardous. The results obtained by these methods can be used for setting up hypotheses, but in any cases, those hypotheses should be verified by deterministic models and/or by direct observations. Therefore, the link between time series analysis methods and the conceptual model of Hobbs & Smart (1986) is almost impossible to establish. One can expect to evidence some characteristics of the subsystems postulated in the conceptual model, meanwhile the dependence on precipitation frequency and the low sensitivity to events intensity make quantitative classification and interpretation impossible.

The global or lumped parameter models (reservoir models) of the hydraulic behaviour of karst systems also lack of spatial predictive power because they do not consider the strong spatial heterogeneity of karst aquifers.

Although quite sophisticated, most of the methods presented here can hardly be used in the interpretation of spring hydrographs in terms of structure of the flow system. Derived inferences will always remain very qualitative and not really more informative than a rough - visual - estimation of the shape of the hydrograph as suggested on figure 1. In some particular cases, they can provide useful results.

As illustrated in the following paper (Laroque et al. 1998, this volume) cross correlation methods can also be used to analyse the relations between other parameters than "precipitation and discharge" such as chemical composition, isotopes or water heads, etc. Meanwhile, interpretation of the transfer function for inferring the structure of the flow system is even more difficult and questionable in such cases. In this particular situation, more information can be obtained when coupling the effects of hydraulics on the parameter variations. Grasso & Jeannin (1998, this volume) show that, even for a trend analysis, this coupling is required because of the predominant influence of the discharge on water chemistry. The coupling of hydraulics with a particular parameter is possible by introducing a deterministic component into a global modelling approach. This type of approach has been developed and quite successfully applied for analysing the thermal response of karst springs (Renner 1996, Renner & Sauter 1997, Liedl et al. 1997) or the variations of calcite carbonate concentrations (Grasso 1998).

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